

Screening Solutions in Modified Gravity Theories

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In this work, we illustrate through a simple example the possibility of testing the chameleon screening mechanism in the Solar System using the forthcoming LISA Pathfinder mission around gravitational saddle points. We find distinctive tidal stress signatures for such models and consider the potential for constraints.

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A. Introduction

In this short letter we aim to show that *chameleonesque* screening mechanisms are naturally present in preferred acceleration modified gravity theories. As such they are of significant interest if we are looking to hide additional effects on Solar System scales and also tests of these modified theories should reach the level of potentially certifying or vilifying models and classes of theories in the very near future [1]. The use of the forthcoming LISA Pathfinder mission [2] provides an opportunity to directly probe an unexplored low acceleration regime as well, as well as test some less conventional ideas in gravitational physics [3, 4]. This presents the opportunity to probe the *onset* of such mechanisms and placing constraints on them in a new and interesting way would be a key result. At the saddle point (SP) of the gravitational field, we find the regime where both $\rho \rightarrow 0$ and $|\nabla\Phi_N| \rightarrow 0$ which we see is reminiscent of the “cosmological regime”.

We can consider as a simple example the Chameleon [5],

$$S = \int \left(\frac{M_{pl}^2}{2} R + X - V(\phi) \right) \sqrt{-g} d^4x + \int \mathcal{L}_m(\Psi_i) \sqrt{-\tilde{g}} d^4x \quad (1)$$

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \quad (2)$$

$$X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \quad (3)$$

EoM :

$$\square\phi = V_{,\phi} - A^3(\phi) A_{,\phi} \tilde{T} = V_{eff,\phi} \quad (4)$$

where we find that $\tilde{T} = -\tilde{\rho}$ and $\tilde{\rho} = A^{-3}\hat{\rho}$ where $\hat{\rho}$ is the matter frame conserved energy density and $\hat{\rho}$ is the Einstein frame conserved energy density such that $\hat{\rho} \neq \tilde{\rho}(\phi)$. This leads us to the relation

$$V_{eff} = V(\phi) + A(\phi)\hat{\rho} \quad (5)$$

This then makes for interesting behaviour in the regime where $\rho \rightarrow 0$ since with a properly chosen V can lead to

quintessence behaviour (such as the Peebles-Ratra potential [6]). Naturally if ρ becomes more prominent, then the effective minima of the potential is shifted and the mass of the field (from $m^2 = \partial^2 V / \partial \phi^2|_{\phi=\phi_{min}}$) becomes much heavier. In typical chameleon models, we need A to take some runaway form such that the mass of the scalar field ϕ becomes too heavy to detect in earth based experiments but on cosmological scales V becomes the dominant contribution allowing it to act as dark energy. Additionally in this mechanism, $V = V(\phi)$ only, otherwise other kinds of screening are present. Originally $A(\phi)$ mechanisms were considered, however these have been generalised to those with derivative screening [7], $A(\phi, X)$.

A Toy Model. We begin with the standard treatment from the TeVeS action [8], neglecting however the vectorial terms (a treatment that we can justify safe in the knowledge that the vector field does not enter into the weak field limit of the theory, so as far as quasi-static systems see, this is the effective theory for TeVeS),

$$S = \frac{1}{2} \int \sqrt{-g} d^4x \left(\frac{R}{8\pi G} - \frac{f}{\kappa G} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{V(f)}{2\kappa \ell^2 G} \right) + \int \mathcal{L}_m(\tilde{g}_{\mu\nu}, f^\alpha, \dots) \sqrt{-\tilde{g}} d^4x \quad (6)$$

where here $\tilde{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu}$, i.e. there exists a fully conformal mapping between the gravity and matter metrics (a feature generically not true with the addition of vector fields here)

From this we find the equations of motion for the metric as the usual Einstein equation with the bimetrically coupled matter

$$G_{\mu\nu} = 8\pi G (\tilde{T}_{\mu\nu} + \tau_{\mu\nu}) \quad (7)$$

$$\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta(\sqrt{-\tilde{g}} \mathcal{L}_m)}{\delta \tilde{g}_{\mu\nu}} \quad (8)$$

$$\tau_{\mu\nu} = \frac{f}{\kappa G} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{V(f)}{f \kappa \ell^2} g_{\mu\nu} \right) \quad (9)$$

additionally we have the ϕ field equation

$$\nabla_\nu \left(\frac{f}{\kappa G} g^{\mu\nu} \partial_\mu \phi \right) = \tilde{T} = \tilde{\rho} e^{-2\phi} \quad (10)$$

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where ∇_μ is the covariant derivative associated with the Einstein frame metric $g_{\mu\nu}$ and we source the stress energy with a pressureless perfect fluid in the matter frame. Finally for the non-dynamical scalar f , we find an equation of motion of the form

$$-\frac{1}{2}\frac{\partial V}{\partial f} = \kappa\ell^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad (11)$$

where ℓ is a length scale related to a_0 . From the expansion of (10),

$$f\Box\phi = \kappa G\tilde{\rho}e^{-2\phi} - g^{\mu\nu}\nabla_\mu f\nabla_\nu \phi \quad (12)$$

where $\tilde{\rho}$ is the matter frame density, related to the Einstein frame conserved density by $\tilde{\rho} = A^{-3}(\phi)\hat{\rho}$. Putting this together and rearranging

$$\Box\phi = (f^{-1}\kappa Ge^{4\phi})\hat{\rho} - \underbrace{f^{-1}g^{\mu\nu}\nabla_\mu f\nabla_\nu \phi}_{-2X\partial f/\partial\phi} \quad (13)$$

$$\Rightarrow = A_{,\phi}\hat{\rho} + V_{,\phi} \quad (14)$$

where it is crucial $V = V(\phi)$ only and we will show how this is achieved later.

B. Recasting A Screening Mechanism as a Preferred Acceleration Theory

First we need to define the order parameter z

$$z = \frac{|\nabla\phi|}{M} \quad (15)$$

and consider the modified Poisson equation

$$\nabla_\mu(f\nabla^\mu\phi) = f\nabla_\mu\nabla^\mu\phi + \nabla_\mu f\nabla^\mu\phi = \kappa G\rho e^{-2\phi} \quad (16)$$

such that for $z \gg 1$, $f \rightarrow 1$. At this stage, we consider dropping the exponential term on the source, citing that we are using a quasi-static approximation here (see Section IIIC of [8] for more details on this step).

Comparing this with the Chameleon equation of motion

$$\nabla_\mu\nabla^\mu\phi - V_{,\phi} = A_{,\phi}\hat{\rho} \quad (17)$$

giving us a clear association between

$$\frac{\partial V}{\partial\phi} = -\frac{1}{f}\nabla_\mu f\nabla^\mu\phi = -\frac{1}{f}\frac{\partial f}{\partial\phi}\nabla_\mu\phi\nabla^\mu\phi = 2X\frac{\partial\ln f}{\partial\phi} \quad (18)$$

$$V = 2X\ln f \iff f = \exp\left(\frac{V}{2X}\right) \quad (19)$$

Recall the Peebles Ratra potential [9]

$$V = C_1^2 \frac{M_{pl}^{n+4}}{\phi^n} \quad (20)$$

making

$$f = \exp\left(-C_1^2 \frac{M_{pl}^{n+4}}{\phi^n |\nabla\phi|^2}\right) \quad (21)$$

so this represents a more general free function in $f(\phi, z)$.

C. An $n = 0$ theory

It is worth therefore associating

$$z = \frac{|\nabla\phi|}{C_1 M_{pl}^2} = \frac{\kappa}{4\pi} \frac{|\nabla\phi|}{a_0} = \frac{|\nabla\phi|}{M} \quad (22)$$

making

$$f = \exp\left(-\frac{1}{z^2}\right) \quad (23)$$

Given we are entering the quasi-static regime with effective equation of motion

$$\nabla \cdot (f\nabla\phi) = \kappa G\tilde{\rho} \quad (24)$$

we will fix

$$\kappa \rightarrow \frac{C_2}{2G} \quad (25)$$

where $C_2/2$ can be thought of as the limiting value of $A(\phi)$ as $\phi \rightarrow 0$, this gives us a definition for a_0 of

$$a_0 = C_1 C_2 M_{pl}^4 \quad (26)$$

These together give the relation

$$f = \exp\left(-\frac{1}{z^2}\right) \quad (27)$$

which obviously satisfies $f \rightarrow 1, z \gg 1$. Such relations then mean that the linear variable choice $U = fz$ will be in the regimes of $U \gg 1 \rightarrow z \gg 1$ and so the bubble size can be inferred from $|U|^2 \simeq 1$

$$z^2 \simeq 1 \Rightarrow \left(\frac{\kappa}{4\pi}\right)^2 \frac{A^2 r^2 N^2}{M^2} \simeq 1$$

$$r^2 |\mathbf{N}|^2 = \left(\frac{4\pi M}{\kappa A}\right)^2 = \left(\frac{C_1}{C_2 A}\right)^2 = r_0^2 \quad (28)$$

For the Earth-Sun SP, this takes the value $r_0 \simeq C_1/C_2 \times 10^8$ km.

1. $z \ll 1$ - Inner Bubble Regime

Here the issue is that f vanishes at $z = 0$ and so there is no expansion we can make here, however since we are unlikely to sample the signal *exactly* at the SP, we can make do with an expansion at small z . The conclusion of which is

$$f \simeq \lim_{p \rightarrow \infty} z^p, \quad z \ll 1 \quad (29)$$

and using the standard tools to compute the form of \mathbf{F}_ϕ [10, 11], we find

$$-\nabla\phi \simeq M \lim_{p \rightarrow \infty} \left(C^{\frac{1}{p+1}} \frac{\mathbf{D}}{D^{\frac{p}{p+1}}} \left(\frac{r}{r_0} \right)^{\frac{1}{p+1}} \right) \quad (30)$$

$$\simeq M (F_0(\psi) \mathbf{e}_r + G_0(\psi) \mathbf{e}_\psi) \quad (31)$$

where we approximate the deep inner bubble solutions with

$$F_0 \simeq 0.024 + 0.886 \cos 2\psi - 0.012 \cos 4\psi \quad (32)$$

$$G_0 \simeq -1.090 \sin 2\psi + 0.022 \sin 4\psi \quad (33)$$

The tidal stresses therefore are

$$S_{yy} \rightarrow \frac{C_1 M_{pl}^2}{2r} S_1(\psi) + \frac{C_2 M_{pl}^2 A}{2} \quad (34)$$

where $S(\psi)$ is computed from the separable ansatz profile functions and the charge of variable and A is the Newtonian tidal stress at the SP. We plot the predicted spatial variations of the tidal stresses in Figure 2 (in arbitrary units), noting both the very different radial dependence as well as the different overall profile function.

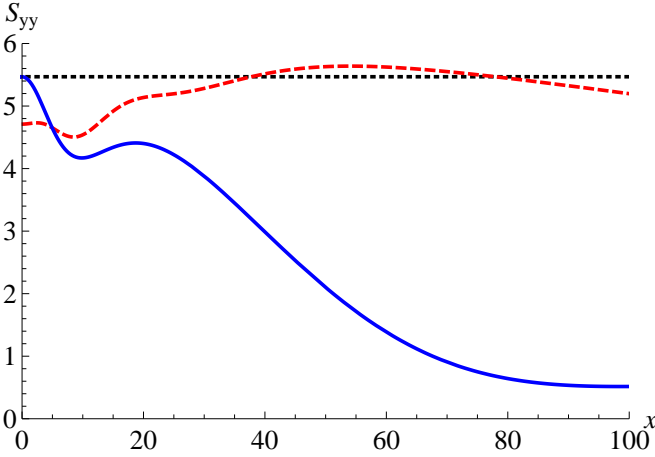


FIG. 1: Comparing the expected spatial variation of tidal stress signals from a typical MONDian signal (red, dashed line), expected Newtonian signal (black, dotted line) and an expected signal from a chameleon (blue, solid line) as potential measured. The axes here label the magnitude of one component of the transverse tidal stress (S_{yy}) in arbitrary units as well as the spatial variation along a trajectory (along the x axes) with a finite miss distance from the SP.

2. $z \gg 1$ - Outer Bubble Regime

In the outer bubble regime, we simply expand f

$$f \simeq 1 - \frac{1}{z^2} + \dots \quad (35)$$

and proceed to find solutions using the standard techniques in these theories. The expected tidal stresses therefore are of the form

$$S_{yy} = \frac{C_1 M_{pl}^2}{2} S_2(\psi) r^{-2} \quad (36)$$

where $S_2(\psi)$ is presented in full in [11].

D. An $n \neq 0$ Theory

The identification there must be made here is

$$f = \exp \left(-C_1^2 \frac{M_{pl}^{n+4}}{|\nabla \phi|^2 \phi^n} \right) \quad (37)$$

$$\phi^n |\nabla \phi|^2 = \left(\frac{2}{n+2} \right)^2 |\nabla \tilde{\phi}|^2 \quad (38)$$

$$\tilde{\phi} = \phi^{1+n/2} \quad (39)$$

$$z = \frac{|\nabla \tilde{\phi}|}{M} \quad (40)$$

$$M = C_1 M_{pl}^{n/2+2} \left(1 + \frac{n}{2} \right) \quad (41)$$

Additionally we see that the bubble boundary is modified

$$|\mathbf{U}|^2 \simeq 1 \Rightarrow |z|^2 \simeq 1 \quad (42)$$

$$\phi^n |\nabla \phi|^2 \simeq M^2 \quad (43)$$

$$r^{2(n+1)} (N_r)^n |\mathbf{N}|^2 \simeq r_0^2 \quad (44)$$

And as such,

$$a_0 = C_1 C_2 M_{pl}^{n/2+4} \left(1 + \frac{n}{2} \right) \quad (45)$$

$$r_0 = \frac{2^n C_1}{(A C_2)^{n+1}} M_{pl}^{-3n/2} \left(1 + \frac{n}{2} \right) \quad (46)$$

The key feature of this result is that it implies regions close to the saddle are generically inside the modified regime.

Here we can follow the same procedure as in Section C1, working with an expression for f

$$f \simeq z^q \quad (47)$$

$$U \simeq z^{q+1} \quad (48)$$

$$f \simeq U^{\frac{q}{q+1}} \quad (48)$$

where ultimately we are taking $q \rightarrow \infty$. Putting this together gives

$$-\nabla \tilde{\phi} \simeq M \frac{\mathbf{D}}{D^{\frac{q}{q+1}}} \left(\frac{r}{r_0} \right)^{\frac{1}{q+1}} \quad (49)$$

which in the large q limit reduces to expression for ϕ in (31). However to recover the actual force from the physical potential ϕ , we first reduce to the potential

$$\tilde{\phi} = -M F_0 r \quad (50)$$

$$\phi = \tilde{\phi}^{\hat{n}} = -(M F_0 r)^{\hat{n}} \quad (51)$$

$$-\nabla \phi = \tilde{M} r^{\hat{n}-1} (F_n(\psi) \mathbf{e}_r + G_n(\psi) \mathbf{e}_\psi) \quad (52)$$

$$(F_n, G_n) = ((F_0)^{\hat{n}}, (G_0)^{\hat{n}}) \quad (53)$$

$$\tilde{M} = C_1^{\hat{n}} M_{pl}^{n+4/n+2} \hat{n}^{-\hat{n}} \quad (54)$$

$$\hat{n} = \frac{2}{n+2} \quad (55)$$

which clearly reduces to (31) for $n = 0$ but here generalises our result. The corresponding tidal stresses therefore are

$$S_{yy} = a_1 S_3(\psi) r^{-c} \quad (56)$$

$$c = \frac{2n+2}{n+2} \quad (57)$$

$$a_1 = \frac{\tilde{M}}{2} \quad (58)$$

where $S_3(\psi)$ is calculated from the change of variable and components of $\nabla\phi$ (more details of which can be found in [11]). These results show that the tidal stresses diverge with radial exponent $1 < c < 2$.

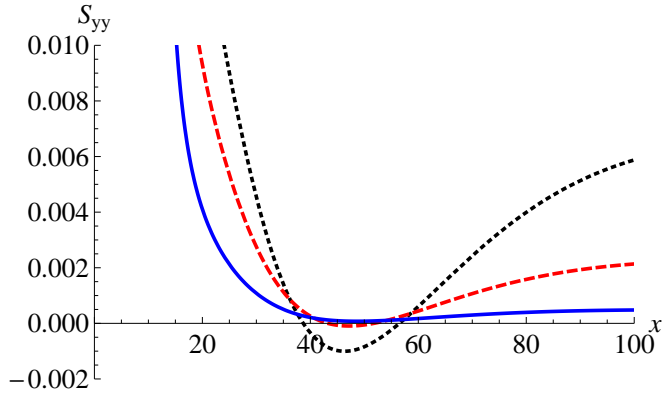


FIG. 2: Comparing the expected spatial variation in tidal stress signals from a typical chameleon signal for $n = 1$ (black, dotted line), $n = 2$ (red, dashed line) and $n = 5$ (blue, solid line).

1. Regime of validity

The next question to ask is how valid our solutions, this points to the validity of the quasi-static regime used to derive our relation (16). We see that if

$$\exp[-2\phi] \simeq 1 \quad (59)$$

then our relation

$$\nabla \cdot (f \nabla \phi) = \frac{\kappa}{4\pi} \nabla^2 \Phi_N \quad (60)$$

holds, the departures from this signal the onset of the full chameleon mechanism and the regime where our results are not applicable. As a rough estimate, we could take this to mean for departures of $|\delta| \sim \mathcal{O}(10^{-1})$

$$\delta \simeq 2MF_0 r \quad (61)$$

$$C_1 r \lesssim 1.7 \times 10^{-10} \quad (62)$$

2. Constraints from data

Different types of constraint are expected to hold for these models:

- **G_N Renormalisation** It is rather bad form to let our effective gravitational constant G vary from G_N by too much, partly because it will mess up the cosmology of such theories. The contribution from ϕ can be seen in the large z limit as

$$G_{eff} = G_N \left(1 + \frac{\kappa}{4\pi}\right) = G_N(1 + C_2 M_{pl}^2) \quad (63)$$

Thus for $|\Delta G| \lesssim 10^{-1}$ [12],

$$C_2 \lesssim 1.7 \times 10^{-10} \quad (64)$$

- **Sensitivity** Given that we have not detected anything like a signal from a fifth force field in the Solar System, it is prudent to imagine that only with new experiments could the possibility of detection become viable. As a naive first consideration, bounds on fifth forces from variations in Kepler's constant and precessions of Mercury and other inner Solar System objects [13]. This gives an upper bound on the size of such forces (along with the expectation that they are “long range”). Thus we argue that they could be hidden within the sensitivity of current measurements, putting a bound on C_1 , of the order of

$$C_1 \lesssim 10^{-15} \quad (65)$$

Given this, the magnitude of the expected tidal stresses are within the range accessible from LPF. If C_1 is drastically smaller than this, this signal will unlikely to be seen above the background and noise (although for the more complicated $n \neq 0$ models with stronger divergences, this remark is subject to change). Taking this idea from the reverse point of view, if no signal is seen, this represents the best constraint on C_1 that we can make.

If we put these together with the quasi-static requirement, this results in

$$r \simeq 1.7 \times 10^4 \text{ m} \quad (66)$$

which is at similar level as the current best estimates for a SP miss with LPF.

E. Conclusions

In this work we develop a test for Chameleon screening mechanisms in the Solar System using the forthcoming LISA Pathfinder mission. We recast such theories in the language of modified gravity theories with a preferred acceleration scale. In doing so we present the expected

tidal stresses for such theories around the gravitational SP in the Solar System, specialising to the Earth-Sun system. Using a combination of analytical results and numerical suggestions we propose that such a test could make it possible to test such theories cleanly, depending upon the precise details of the models used. In a forthcoming paper [14], we will expand on our methods as well as focus on other screening mechanism and the prospects for observation.

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